

Praxis-Logos Imbalance in Pythagorean Theorem Textbook Content: A Praxeological Diagnosis of Indonesia's Kurikulum Merdeka for Mathematics Education Quality

Rizqi Amaliyakh Sholikhakh¹, Isnani¹, Agus Hendriyanto², Lukman Hakim Muhaimin²

Department of Mathematics Education, Faculty of Teacher Training and Education, Pancasakti University Tegal, Tegal, Indonesia¹

Department of Master's Program in Pedagogy, Nusa Putra University, Sukabumi, Indonesia²

E-mail Corresponding: rizqias@upstegal.ac.id

Received: June 15, 2026

Revised: August 10, 2026

Accepted: September 21, 2026

Abstract

Mathematics textbooks function as primary didactic artifacts that shape the organization of mathematical knowledge in classroom settings. This study aims to analyze the praxeological organization of the Pythagorean Theorem content in a Grade VIII mathematics textbook for lower secondary education under the Kurikulum Merdeka, the analyzed textbook is *Matematika untuk SMP/MTs Kelas VIII (Semester 2)*, published by Kemendikbudristek (ISBN 978-602-427-919-6); the analysis focused on Chapter 2 (The Pythagorean Theorem, pp. 46–96) using the framework of the Anthropological Theory of the Didactic developed by Yves Chevallard. The research employs a qualitative content analysis method, with units of analysis consisting of task types (T), techniques (τ), technology (θ), and theory (Θ) identified across five subchapters of the Pythagorean Theorem. The findings reveal 28 task types organized into eight scaffolding stages, with a significant dominance of the praxis block over the logos block. Based on descriptive coding of task categories (not statistical analysis), approximately 70 percent of the tasks are procedural and computational in nature, while only 15 percent require reasoning and proof. The technology (θ) that justifies techniques is predominantly implicit, and the theory (Θ) that legitimizes the technology is entirely absent in explicit form. Further analysis identifies 16 potential learning obstacles, including epistemological, didactic, and ontogenetic obstacles. These findings suggest the need to strengthen the logos block through the inclusion of alternative proofs, explicit justification of techniques, and diversification of task types to support the development of students' relational understanding. This study contributes to a deeper understanding of how didactic transposition shapes praxeological organization in textbooks and its implications for the quality of mathematics learning.

Keywords: Anthropological Theory of the Didactic; Pythagorean Theorem; textbook analysis, Kurikulum Merdeka; Learning obstacles

INTRODUCTION

Mathematics textbooks occupy a central position within the didactic ecology as the primary mediators between mathematical knowledge produced by the scientific community, known as *savoir savant*, and the knowledge taught in the classroom, referred to as *savoir enseigné*. In the perspective of the Anthropological Theory of the Didactic (ATD) developed by (Chevallard, 1999; Lukman Hakim Muhaimin et al., 2026; Winsløw, 2011), a textbook is a product of didactic transposition that reconstructs mathematical knowledge into a specific praxeological organization suitable for teaching. This praxeological organization consists of four interrelated components: types of tasks (T), techniques (τ), technology (θ), and theory (Θ). Together, these elements form what Chevallard defines as a praxeology. The first two components constitute the praxis block, representing the "know how" aspect of mathematical knowledge, while the latter two form the logos block, representing the "know why" aspect (Chevallard, 1989; Inhelder & Piaget, 1958; L. H. Muhaimin et al., 2023).

The praxeological analysis of mathematics textbooks has become increasingly vital in the context of curriculum reform in Indonesia, particularly with the implementation of Kurikulum Merdeka. This curriculum promotes a constructivist paradigm oriented toward the development of 21st century competencies. Kurikulum Merdeka emphasizes meaningful learning that moves beyond rote memorization of procedures toward building deep conceptual understanding (Kemendikbudristek, [2022](#)). Nevertheless, the extent to which textbooks, as primary didactic artifacts, successfully translate this curricular vision into a balanced praxeological organization between procedural aspects and reasoning remains a question requiring rigorous empirical investigation.

The Pythagorean Theorem is one of the most fundamental mathematical topics in the junior high school curriculum. Beyond its historical significance as one of the oldest theorems in mathematics, it serves a strategic role as a bridge between geometry and algebra (Florio, [2020](#)). A profound understanding of the Pythagorean Theorem requires the integration of geometric area concepts, algebraic manipulation, deductive reasoning, and mathematical modeling skills (L. H. Muhaimin et al., [2025](#); Wiranugraha et al., [2025](#)). Consequently, the way textbooks present this topic, including how tasks are designed, which techniques are privileged, the extent to which techniques are justified, and the theory underlying the entire organization, serves as a critical indicator of the didactic quality of the textbook.

The ATD framework has been widely utilized in international mathematics education research to analyze the organization of knowledge in textbooks and instructional practices. Hardy ([2009](#)) applied ATD to analyze the teaching of limits of functions in Spain and discovered that classroom praxeological organizations tended to be dominated by the praxis block, leaving the logos block fragmented. Similar findings were reported by Wijayanti & Winslow ([2017](#)) in their analysis of trigonometry textbooks in Denmark and Indonesia, which indicated that Indonesian textbooks frequently privilege algorithmic techniques with minimal justification. Research by González-Martín et al. ([2014](#)) regarding the teaching of integrals across various countries confirmed that the imbalance between praxis and logos is a phenomenon that transcends cultures and topics. Furthermore, Chaachoua & Comiti ([2010](#)) demonstrated that praxeological analysis can reveal hidden epistemological assumptions within textbooks that have significant implications for the quality of student understanding.

In the specific context of teaching the Pythagorean Theorem, several studies have identified various didactic challenges. Zazkis & Leikin ([2008](#)) showed that proofs of the Pythagorean Theorem in textbooks are often ritualistic and disconnected from problem solving activities. Huang & Leung ([2002](#)) compared the presentation of the theorem in Chinese and Hong Kong textbooks, finding significant differences in praxeological organization, particularly in the proportion of proof tasks versus computational tasks. Research by Clements & Battista ([1992](#)) suggested that the visual orientation of right triangles in textbooks significantly influences the formation of students' concept images, where the dominance of standard orientations can hinder the recognition of right triangles in non standard contexts. Additionally, Fischbein et al. ([1995](#)) identified that irrational numbers resulting from the application of the theorem constitute a fundamental epistemological obstacle that is rarely addressed adequately by textbooks (Fischbein et al., [1995](#)).

In Indonesia, research regarding the praxeological analysis of Kurikulum Merdeka textbooks remains very limited. While some studies have analyzed the pedagogical aspects of these textbooks in general (Hendriyanto et al., [2023](#); Lukman Hakim Muhaimin et al., [2026](#)), in depth analysis using the ATD framework focused on the praxeological organization of specific content is still rare. This is noteworthy given that Indonesia has undergone several curriculum changes over the past two decades, moving from the Competency Based Curriculum to the 2006 Curriculum, the 2013 Curriculum, and finally to Kurikulum Merdeka. Each transition brought changes to how mathematical knowledge is organized in textbooks. Analyzing the Kurikulum Merdeka textbooks is essential to evaluate whether the didactic transposition has produced a praxeological organization aligned with the curricular vision of conceptual understanding and

mathematical reasoning.

The theory of learning obstacles developed by Brousseau (1997) provides an important complementary framework for textbook analysis. Brousseau classifies learning obstacles into three categories: epistemological obstacles related to the nature of the knowledge itself, didactic obstacles arising from instructional choices, and ontogenetic obstacles related to the students' cognitive development. Identifying potential learning obstacles emerging from the praxeological organization of a textbook is a crucial step in designing appropriate didactic interventions. Duval (2006) added that conversion between registers of representation, such as moving from geometric to algebraic representations, is a major source of difficulty in mathematics learning that is often not sufficiently anticipated in textbook design.

Moreover, Artigue (2002) emphasized that the dialectic between technical work and conceptual work is a fundamental aspect of mathematics learning that is frequently mismanaged in textbook design. In the context of the Pythagorean Theorem, this dialectic is manifested in the relationship between procedural calculations, such as finding side lengths, and conceptual understanding, such as why the squared relationship only applies to right triangles. Tall & Vinner (1981) introduced the concepts of concept image and concept definition which are relevant here. Students often possess a concept image of the Pythagorean Theorem limited to the formula $c^2 = a^2 + b^2$ without a concept definition that encompasses geometric relationships with area and the requirement of a right angle. Praxeological analysis can uncover the extent to which a textbook supports or limits the formation of a rich and accurate concept image.

This study is novel in three respects. First, it provides the first praxeological (ATD-based) analysis of Pythagorean Theorem content in an official Grade VIII Kurikulum Merdeka mathematics textbook, addressing a topic and curriculum context not previously examined through this lens. Second, it integrates praxeological organization analysis with Brousseau's (1997) theory of learning obstacles, yielding a comprehensive didactic diagnosis rather than a purely structural description. Third, it systematically quantifies the praxis-logos balance through transparent descriptive coding of discrete task items, offering an auditable, evidence-based account of the imbalance. Collectively, these contributions position the study within the ISEJ scope by linking textbook quality, mathematical reasoning development, and Kurikulum Merdeka curriculum implementation.

The research gap identified from this literature review indicates that a comprehensive and deep praxeological analysis of Kurikulum Merdeka textbooks, specifically concerning the Pythagorean Theorem, has not yet been conducted. Existing research has not integrated the analysis of praxeological organization with the identification of potential learning obstacles arising from the characteristics of that organization. Therefore, this study aims to address three research questions. First, how is the praxeological organization of the Pythagorean Theorem structured in the Grade VIII Kurikulum Merdeka mathematics textbook in terms of task types, techniques, technology, and theory? Second, what is the balance and interconnection between the praxis and logos blocks within that organization? Third, what learning obstacles potentially emerge from the identified characteristics of the praxeological organization?

This research holds both theoretical and practical significance. Theoretically, it contributes to the understanding of how didactic transposition operates within the Indonesian curricular context and how the balance of praxis and logos is manifested in national textbooks. Practically, the findings can provide valuable feedback for textbook developers, teachers, and curriculum designers to improve the quality of the praxeological organization of the Pythagorean Theorem and other mathematical topics. In line with the scope of ISEJ, these findings directly inform the quality of mathematics and science education by clarifying how the design of textbook tasks shapes opportunities for reasoning development and by offering actionable guidance for curriculum implementation under Kurikulum Merdeka.

METHODS

This study employed a qualitative approach using content analysis to examine a mathematics textbook. Content analysis was selected as it enables researchers to systematically and deeply identify and interpret the organization of knowledge embedded within textual materials (Krippendorff, [2018](#)). This approach aligns with the tradition of textbook analysis in mathematics education research, which has been widely adopted by researchers working within the Anthropological Theory of the Didactic framework (Chevallard, [2019](#); Winsløw, [2011](#)).

The data source of this study was a Grade VIII mathematics textbook for lower secondary education published by the Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia, with ISBN 978-602-427-919-6 (Volume 2) (Kemendikbudristek ([2022](#)), *Matematika untuk SMP/MTs Kelas VIII, Semester 2* (First Edition), Jakarta: Pusat Perbukuan, Badan Standar, Kurikulum, dan Asesmen Pendidikan, Kemendikbudristek, ISBN 978-602-427-919-6 (Volume 2)). This textbook serves as an official national resource for the implementation of Kurikulum Merdeka. The unit of analysis focused on Chapter 2: The Pythagorean Theorem, which spans 52 pages (pages 46 to 96) and is organized into five subchapters: (A) Discovering the Concept of Pythagoras, (B) Pythagorean Triples, (C) Special Triangles, (D) Applications of the Pythagorean Theorem, and (E) Distance Formula as enrichment material, complemented by a competency test section. This single textbook was purposively selected because it is the officially mandated national resource distributed for the implementation of Kurikulum Merdeka, making it the most representative artifact of the intended curriculum; accordingly, the findings characterize the praxeological organization of one chapter of this nationally used textbook rather than broader cross-textbook tendencies.

The analytical framework applied in this study was the praxeological organization from the Anthropological Theory of the Didactic developed by Chevallard ([1999](#)). Within this framework, each unit of mathematical knowledge is analyzed through four components forming a praxeology [T, τ , θ , Θ]. Types of tasks (T) refer to categories of problems or questions that students are expected to solve, for example determining the length of a hypotenuse or verifying whether three numbers form a Pythagorean triple. Techniques (τ) are the procedures or methods used to solve a given type of task, such as substituting values into the formula $c^2 = a^2 + b^2$ followed by computing the square root. Technology (θ), in ATD terminology, refers to the discourse that justifies and explains why a technique works rather than technology in the common sense. An example is the formal proof of the Pythagorean Theorem that explains the validity of the formula. Theory (Θ) represents the broader theoretical framework that legitimizes the technology and ensures coherence of the overall organization, such as Euclidean geometry as the axiomatic system underlying the entire construction (Bosch & Gascón, [2006a](#)).

The analysis procedure consisted of four stages. The first stage involved identifying types of tasks (T), in which all exercises, examples, and activities in the chapter were examined and categorized based on the type of problem presented. Each task type was coded from T1 to T28 and described in detail. The second stage focused on identifying techniques (τ), namely the solution procedures privileged by the textbook for each task type. Techniques were analyzed according to their characteristics, including algorithmic, empirically inductive, strategic, or multi step procedures. The third stage involved analyzing technology (θ) by identifying the justifications and explanations underlying each technique. The level of explicitness of technology was categorized into explicit, semi explicit, and implicit. The fourth stage examined theory (Θ), identifying the broader theoretical frameworks that legitimize the technology and provide coherence to the overall praxeological organization.

In addition to analyzing individual components, an interconnection analysis was conducted to evaluate how the four components support or disconnect from each other. The balance between the praxis block [T, τ] and the logos block [θ , Θ] was analyzed qualitatively and supported by calculating the proportion of tasks based on their orientation, including procedural computational, reasoning and proof, or exploratory tasks. The identification of potential learning obstacles was guided by Brousseau ([1997](#)) classification, which distinguishes epistemological,

didactic, and ontogenetic obstacles.

Following the identification of the four components, a second level analysis was conducted covering three aspects. First, scaffolding analysis was performed to identify the sequence and progression of task types across subchapters. Second, analysis of the connectivity between techniques and technology was conducted to evaluate the extent to which each technique was supported by adequate justification. Third, analysis of the balance between praxis and logos was carried out to examine the proportion between procedural computational activities and reasoning or proof activities. The analysis of learning obstacles was conducted by identifying features of the praxeological organization that may potentially generate learning difficulties based on Brousseau's classification.

The validity of the analysis was ensured through three strategies. First, theoretical triangulation was employed by combining the ATD framework and Brousseau's theory of learning obstacles to produce richer and more reliable interpretations. Second, iterative analysis was conducted through repeated readings of the textbook, with a minimum of three analytical cycles to ensure completeness in identifying praxeological components and consistency in categorization. Each cycle focused on a different aspect: the first cycle on identifying task types and techniques, the second on analyzing technology and theory, and the third on examining interconnections and learning obstacles. Third, peer debriefing was carried out by discussing the analytical findings with mathematics education experts to verify interpretations and categorizations and to identify potential biases in the analysis.

A formal coding protocol governed the analysis. Two coders with expertise in mathematics education (the first and second authors) independently coded all discrete task items using the praxeological categories [T, τ , θ , Θ]. The explicitness of technology (θ) was operationalized on three levels: explicit (a complete logical justification of why a technique works is provided), semi-explicit (a partial or incomplete justification is provided), and implicit (a rule or technique is stated without any justification). Task types were categorized as procedural-computational (direct application of a known procedure or formula), exploratory (open investigation, conjecturing, or discovery), or reasoning-and-proof (tasks requiring justification, generalization, or proof).

To ensure the rigor and confirmability of the coding, the two coders first coded a common subset independently, and inter-coder reliability was computed using Cohen's Kappa, yielding $\kappa = 0.87$, which indicates a strong level of agreement. All disagreements were resolved through consensus discussion, and a third author with expertise in the Anthropological Theory of the Didactic served as an expert auditor to review the final coding scheme. An audit trail of coding decisions and a peer-debriefing log were maintained to support the confirmability of the analysis.

The reported percentages are the result of descriptive qualitative coding rather than statistical inference. The unit of counting was the discrete task item (each distinct exercise, worked example, and activity prompt). A total of $N = 96$ discrete task items were coded across the five subchapters. Percentages were computed as the count within each category divided by this total: approximately 70% (procedural-computational), 15% (exploratory), and 15% (reasoning-and-proof). For example, computing the hypotenuse from two known legs (p. 54) was coded as procedural-computational, investigating the area relationship on a grid (p. 50) as exploratory, and the algebraic dissection proof (p. 52) as reasoning-and-proof.

RESULTS AND DISCUSSION

Results

Description of the Praxeological Organization

The praxeological analysis of Chapter 2 on the Pythagorean Theorem identified a total of 28 task types, labeled T1 to T28, distributed across five subchapters. These task types exhibit considerable diversity, ranging from prerequisite operations such as squaring and square roots, conceptual discovery through concrete manipulation, direct computation using formulas,

verification and classification of triangles, derivation of ratios in special triangles, to applications in real life contexts and analytic geometry. The distribution of task types reveals a structured scaffolding pattern progressing from concrete to abstract and from simple to complex, reflecting the constructivist approach promoted by the Kurikulum Merdeka. Table 1 summarizes the praxeological organization in each subchapter, including the number of task types, dominant techniques, quality of technology and theory, and the balance between the praxis and logos blocks. The complete coding matrix for all 28 task types (T1–T28), including the technique, technology status, theory status, textbook page reference, and an evidence excerpt for each, is provided in Appendix A to support full auditability of the analysis.

Table 1. Summary of Praxeological Organization by Subchapter

Subchapter	Number of Task Types	Dominant Technique	Quality of θ	Quality of Θ	Balance
A: Concept of the Pythagorean Theorem	8 (T ₁ –T ₈)	Procedural algorithmic	Strong (1 formal proof)	Implicit	Praxis >> Logos
B: Pythagorean Triples	5 (T ₉ –T ₁₃)	Algorithmic verification	Weak (without converse proof)	Implicit	Praxis >> Logos
C: Special Triangles	7 (T ₁₄ –T ₂₀)	Proportional comparison	Moderate (partial derivation)	Semi-explicit	Praxis > Logos
D: Applications	4 (T ₂₁ –T ₂₄)	Modeling and computation	Weak (implicit assumptions)	Implicit	Praxis >> Logos
E: Distance Formula	4 (T ₂₅ –T ₂₈)	Formula substitution	Strong (clear derivation)	Semi-explicit	Praxis > Logos

The textbook adopts a constructivist approach, as reflected in the use of various pedagogical features, including “Let Us Recall” for activating prerequisite knowledge, “Let Us Explore” for concept discovery, “Let Us Try” for guided practice, “Let Us Think Critically” and “Let Us Think Creatively” for developing higher order reasoning, “Let Us Communicate” for mathematical discussion, “Let Us Collaborate” for cooperative learning, and “Let Us Reflect” for metacognition. This variety of features represents an effort to engage multiple dimensions of student competence in alignment with the objectives of the Kurikulum Merdeka. However, the praxeological analysis indicates that these features do not always correspond to the depth of the logos organization. Several activities labeled as “Let Us Think Critically” remain procedural and computational in nature.

As presented in Table 1, Subchapter A, which focuses on discovering the concept of Pythagoras, is the most comprehensive section, comprising eight task types that include prerequisite material, conceptual discovery, formal proof, and application. This subchapter presents a formal proof of the Pythagorean Theorem using an algebraic dissection method that compares two arrangements of four congruent right triangles within a square of side length $(a + b)$. This proof belongs to the category of geometric algebraic proofs and is relatively sophisticated for the junior secondary level. It is also the only formal proof presented in the entire chapter. Subchapter B introduces five task types related to the converse of the Pythagorean Theorem and Pythagorean triples, including algorithmic verification and generative techniques. Subchapter C presents seven task types involving the derivation of ratios in special triangles and the construction of the Theodorus spiral. Subchapter D focuses on four contextual task types that require mathematical modeling skills, while Subchapter E includes four task types related to the distance formula in the Cartesian plane.

An important finding in Subchapter C is that the technology component for special triangles suggests relatively higher quality compared to other subchapters. The ratio 1: 1: $\sqrt{2}$ for isosceles right triangles is explicitly derived using the Pythagorean Theorem, allowing students to understand that the ratio is not a rule to be memorized but a logical consequence of the theorem. However, for the 30° – 60° – 90° triangle, the derivation of the ratio 1: $\sqrt{3}$: 2 is assigned as a task through a “Let Us Think Creatively” activity, supported only by a visual hint involving a bisected equilateral triangle, without explaining why the equilateral triangle is

relevant. In addition, the Theodorus spiral, which connects the Pythagorean Theorem to the sequence of irrational numbers such as $\sqrt{2}, \sqrt{3}, \sqrt{4} = 2$ up to $\sqrt{17}$, is presented as an engaging collaborative activity. This provides a valuable historical and epistemological context that enriches students' understanding, although the historical significance of this construction is not elaborated in the textbook.

Analysis of the Praxis Components

The analysis of the praxis block indicates that the textbook presents a relatively rich variety of task types supported by a structured scaffolding organized into eight stages of progression. These stages include: (1) prerequisites involving operations with squares and square roots; (2) discovery through hands on exploration and numerical verification; (3) formalization in the form of the theorem and formal proof; (4) basic application through direct computation; (5) advanced application involving composite figures and contextual problems; (6) extension through the converse of the theorem and Pythagorean triples; (7) specialization in the form of special triangles; and (8) generalization through real life applications and the distance formula. This progression reflects a logical didactic sequence from concrete experiences to abstract understanding. Table 2 presents an analysis of the techniques that are privileged for the main clusters of task types, along with their characteristics and levels of complexity.

Table 2. Privileged Engineering Analysis for the Main Task Type Cluster

Task Type Cluster	Technique (τ)	Characteristics	Complexity
T ₁ –T ₂ : Prerequisite	Multiplying numbers by themselves; finding square roots	Arithmetic procedural; single-step	Low
T ₃ –T ₄ : Concept discovery	Concrete manipulation (cutting and pasting) and visual observation	Empirical inductive; hands-on	Medium
T ₅ –T ₈ : Pythagorean computation	Substitution into the formula ($c^2 = a^2 + b^2$); shape decomposition; context mathematization	Algorithmic; multi-step	Medium–High
T ₉ –T ₁₃ : Verification and triples	Compare (c^2) with ($a^2 + b^2$); use generators ($(p^2 - q^2, 2pq)$)	Verification algorithmic; generative	Medium
T ₁₄ –T ₂₀ : Special triangles	Ratio comparison of fixed sides ($1:1:\sqrt{2}$) and ($1:\sqrt{2}:2$); derivation from Pythagoras	Proportional reasoning	Medium
T ₂₁ –T ₂₈ : Applications and distance	Modeling positions as coordinates; substitution into the distance formula	Multi-step; modeling and computation	High

The quantitative analysis of all exercises and activities in the chapter indicates that approximately 70 percent of the tasks are procedural and computational, requiring students to apply formulas in familiar situations. Around 15 percent of the tasks are exploratory, involving discovery-oriented activities, and only about 15 percent require reasoning or proof. This distribution reflects the dominance of algorithmic techniques within the textbook's praxeological organization. Although the variety of task types is relatively rich, the privileged techniques remain centered on formula substitution and computation, resulting in a much greater emphasis on the procedural dimension of mathematics compared to the reasoning dimension.

The strength of the praxis block in this textbook lies in its structured scaffolding and meaningful contextualization. The use of Indonesian contexts such as the Kanor Rengel bridge, roof frame constructions, and the traditional game bentengan provides cultural relevance aligned with the principles of Realistic Mathematics Education (Treffers, 1993). Contextual problems in Subchapter D also demonstrate an increase in technical complexity, moving from direct substitution to multi step modeling that requires students to translate verbal information into geometric representations. For example, Example 2.6 involving tourist and fishing boats and Example 2.7 involving a fighter aircraft and an aircraft carrier require students to identify right triangle structures within real contexts, translate verbal descriptions into diagrams, and only

then apply the Pythagorean Theorem. This represents a significant increase in cognitive demand compared to direct computational tasks.

However, a key limitation identified is the lack of variation in techniques for the same task type. Most task types are associated with a single technique, which limits students' procedural flexibility (Kilpatrick et al., 2001). For instance, in determining the length of the hypotenuse, the only technique presented is direct substitution into the formula $c^2 = a^2 + b^2$. Alternative techniques such as using ratios of special triangles when applicable, estimating using nearby Pythagorean triples, or employing geometric construction approaches are not introduced. This limitation reduces students' ability to select the most efficient strategy based on the characteristics of a problem. Furthermore, the analysis of the competency test suggests that multiple choice questions are predominantly direct computational tasks, while essay questions, although more complex, still focus on applying learned procedures without requiring critical evaluation or creative reasoning.

Analysis of the Logos Components

The analysis of the logos block reveals findings that stand in contrast to the richness of the praxis block. The technology component (θ), which serves to justify techniques, shows significant variation in its level of explicitness across subchapters. In Subchapter A, the strongest form of technology is the formal proof of the Pythagorean Theorem using an algebraic dissection method that compares two arrangements of four congruent right triangles within a square of side length $(a + b)$, yielding the equality $c^2 = a^2 + b^2$. This proof is the only formal proof presented in the entire chapter. Although it is of good quality, its singular presentation without alternative proofs limits students' appreciation of the richness of mathematical justification. Research by Leikin (2014) indicates that presenting multiple proofs enhances flexibility of understanding and fosters appreciation for the beauty of mathematics.

Other forms of supporting technology include empirical verification through hands on activities, such as verifying the Pythagorean relationship for specific cases, the derivation of the ratio $1:1:\sqrt{2}$ for isosceles right triangles, and the derivation of the distance formula in the Cartesian plane. These inductive empirical justifications have pedagogical value in building intuition prior to formalization. However, from an epistemological perspective, empirical verification does not constitute proof (de Villiers, 2005). The textbook does not consistently distinguish between verification, which is inductive and case based, and proof, which is deductive and general. This may lead students to develop empirical proof schemes (Sowder & Harel, 1998).

Several significant technological gaps are identified in the analysis. First, the converse of the Pythagorean Theorem is stated without proof. From a mathematical standpoint, the converse of a theorem is not automatically true, and the fact that the converse of the Pythagorean Theorem holds is a non trivial result requiring its own justification. Second, the criteria for classifying triangles, such as $c^2 > a^2 + b^2$ for obtuse triangles and $c^2 < a^2 + b^2$ for acute triangles, are only verified empirically through limited experimental cases without theoretical explanation. Third, the generator formula for Pythagorean triples, namely $(p^2 + q^2, p^2 - q^2, 2pq)$, is presented without algebraic proof that it indeed produces valid triples, even though such a proof is relatively simple and accessible at the junior secondary level. Fourth, properties of square roots are introduced as accepted rules without justification, which in ATD terminology represents implicit technology in the form of rules without explanatory discourse.

The theory component (Θ) exhibits a more fundamental weakness. There is no explicit reference to broader theoretical frameworks that legitimize the technology and provide coherence to the praxeological organization. Euclidean geometry as the axiomatic system underlying the properties of right triangles appears only implicitly. The theory of geometric measurement, which connects area concepts to side lengths, emerges in exploratory activities but is not formalized. Algebraic structures and the real number system, including irrational numbers that arise from square roots, are not explicitly articulated. The Theodorus spiral,

presented as enrichment material, actually provides an appropriate historical and epistemological context for introducing irrational numbers and their significance in the foundational crisis of mathematics among the Pythagoreans, yet this potential is not fully utilized in the textbook.

The absence of explicit theory prevents students from viewing the Pythagorean Theorem as part of a larger and coherent mathematical structure, instead presenting it as isolated knowledge. As noted by Bosch et al. (2006), this leads to the fragmentation of knowledge into disconnected units. Bosch & Gascón (2006) emphasize that without an explicit theoretical framework, a praxeological organization loses both its coherence and epistemological legitimacy, reducing mathematical knowledge to a collection of fragmented procedures without a unifying conceptual foundation.

It is important to clarify that identifying the absence of an explicit theory (Θ) block does not imply that a formal axiomatic treatment of Euclidean geometry should be expected at the Grade VIII level. A distinction must be drawn between formal axiomatic theory, which is developmentally inappropriate for 13–14-year-old students, and age-appropriate theoretical discourse, which is both attainable and desirable. The latter includes explicit verbal justifications ("explain why this works"), visual and geometric proofs accessible to lower-secondary students, guided discovery of why the converse holds, and deliberate connections between the theorem and the area concepts students already possess. Strengthening the logos block at this level therefore means enriching age-appropriate theoretical discourse, not importing formal axiomatics.

Alignment of Praxeological Components

The analysis of the interconnections among the four praxeological components reveals a complex pattern. Table 3 presents an evaluation of the quality of connectivity between techniques and technology for the main techniques identified.

Table 3. Evaluation of Engineering and Technology Connectivity

Technique (τ)	Supporting Technology (θ)	Connection Quality	Notes
Substitution into ($c^2 = a^2 + b^2$)	Algebraic dissection proof	Weak	The proof is presented separately; students apply the formula without referring to the proof
Verification of the converse theorem	Converse theorem (without proof)	Very weak	The technique is justified only by a statement without proof
Triangle classification	Criterion (c^2) vs ($a^2 + b^2$) (empirical)	Weak	Verified only for three cases; generalization is made without proof
Side ratio comparison in 45° – 45° – 90° triangles	Derivation from the Pythagorean theorem	Strong	The technique is directly derived from Pythagorean calculations
Side ratio comparison in 30° – 60° – 90° triangles	Proof task assigned to students	Moderate	The justification is left to students without confirmation
Distance formula between two points	Derivation from the Pythagorean theorem on the Cartesian plane	Strong	The derivation is complete and clearly explained

Table 3 indicates that among the six main techniques analyzed, only two demonstrate strong connections with their supporting technology, namely the derivation of side ratios in 45° – 45° – 90° triangles and the distance formula. Three techniques exhibit weak or very weak connections, meaning that these techniques are applied by students without sufficient understanding of why they are valid. This phenomenon reflects what Chevallard (2006) criticizes as “monumentalism” in mathematics education, where knowledge is presented as a monument to be visited and accepted rather than as an intellectual tool to be understood and justified.

Overall, the imbalance between the praxis and logos blocks is systematic and consistent

across all subchapters. The strength of the praxis block is reflected in the diversity of task types, with 28 types identified, structured scaffolding, and meaningful contextualization. In contrast, the weakness of the logos block is evident in the presence of only one formal proof in the entire chapter, three explicit justifications, the predominance of implicit technology, and the complete absence of explicit theory. Quantitative data show that approximately 70 percent of tasks are procedural and computational, requiring only the application of formulas at the applying level of Bloom's taxonomy, while only about 8 percent of tasks require analysis, evaluation, or creativity at higher cognitive levels. This imbalance contributes to the development of instrumental understanding (Skemp, 1976), which focuses on knowing how without understanding why, and limits the development of higher order thinking skills.

Further analysis of task scaffolding reveals several important gaps. The first gap occurs between hands on exploratory activities, such as cutting and rearranging squares, and formal proof. There is no intermediate step that supports students in articulating their findings verbally before transitioning to algebraic formalization. The second gap concerns the isolation of proof. The formal proof is presented between the statement of the theorem and computational exercises but is not revisited in subsequent tasks, so students are never required to apply proof based reasoning in problem solving. The third gap appears in the transition from the Pythagorean Theorem to its converse in Subchapter B, which is introduced without proof, creating a significant logical discontinuity in the mathematical discourse. These gaps indicate that while task scaffolding is well structured at the level of praxis, scaffolding at the level of logos requires further strengthening.

The coherence of the mathematical discourse also reveals notable findings. On the one hand, the textbook presents consistent contextualization through the theme of construction, a logical progression of complexity, and the integration of digital tools such as GeoGebra and coding. On the other hand, several inconsistencies are identified. These include the interchangeable use of the terms "Pythagorean Postulate" and "Pythagorean Theorem" without clarification that they are synonymous, the lack of a clear distinction between empirical verification and deductive proof, and the presentation of the converse theorem without proof, which introduces logical inconsistency into an otherwise structured mathematical discourse.

Discussion

The findings of this study can be contextualized within several complementary theoretical frameworks. From the perspective of the Anthropological Theory of the Didactic, the dominance of the praxis block over the logos block reflects what Bosch et al. (2006) describes as a consequence of didactic transposition, in which mathematically rich epistemological knowledge is reduced to a set of procedures considered teachable and assessable. This finding is consistent with Barbé et al. (2005), who reported fragmentation of the logos block in the teaching of limits in Spain, and with Wijayanti & Winslow (2017), who found a privileging of algorithmic techniques with minimal justification in Indonesian trigonometry textbooks. This suggests that the imbalance between praxis and logos is a structural phenomenon that transcends specific topics and national contexts.

The sequencing of proof presentation in the textbook, beginning with empirical exploration followed by the statement of the theorem, then formal proof, and finally procedural exercises, reflects a pattern that Sowder & Harel (1998) classify as an approach that may lead to the development of an authoritative proof scheme in students. In this pattern, proof is presented as a formal ritual that follows the statement of a theorem but is not revisited in subsequent problem-solving activities. Students learn that formulas can be used not because they understand the logic of the proof, but because their validity has been asserted by the textbook or the teacher. Chevallard & Bosch (2020) notes that within didactic transposition, proof often loses its epistemological function and becomes ritualistic, taking the form of an inserted proof that is placed between statement and application without emerging from an intellectual need.

The phenomenon of incomplete thematization, as conceptualized by Chevallard, is clearly

evident in several parts of the textbook. In Subchapter B, for example, the concepts of the converse of the Pythagorean Theorem and Pythagorean triples are introduced and practiced through exercises but are not fully theorized. Students learn how to verify whether three numbers form a Pythagorean triple but do not understand why the criterion holds. Skemp (2020) distinguishes between instrumental understanding, which involves knowing rules without understanding reasons, and relational understanding, which involves knowing both rules and their underlying rationale. The praxeological organization of the textbook, characterized by a strong praxis block and a weak logos block, tends to promote instrumental understanding. An interesting feature of the textbook is the inclusion of a “Let Us Use Technology” activity that introduces coding to verify Pythagorean triples, reflecting an innovative integration of digital technology. However, its function remains limited to computational verification and does not enhance conceptual understanding of why the generator formula produces valid triples.

From a didactic perspective, comparison with optimal approaches recommended in the literature highlights several areas for improvement. Bell (1976) and Leikin (2014) advocate the use of multiple proofs to enhance appreciation of mathematical depth and beauty. Van Hiele (1986) emphasizes the importance of deep geometric exploration prior to algebraic formalization. Mason et al. (2009) stress the systematic use of counterexamples to strengthen concept formation. Laborde (2001) demonstrates the potential of dynamic geometry software for exploratory learning beyond static verification. While the analyzed textbook adopts some of these recommendations, such as meaningful contextualization, exploratory activities, and integration of GeoGebra, it does not fully implement others, including multiple proofs, systematic use of counterexamples, and deeper geometric exploration prior to algebraic treatment.

These patterns are consistent with recent evidence from the last five years. Florio (2020) argues that reconnecting the Pythagorean Theorem to its historical-conceptual roots can restore the reasoning dimension that procedural presentations tend to suppress, while Skemp (2020) reiterates that an overemphasis on instrumental understanding constrains the development of relational understanding. Within the Indonesian context, Hendriyanto et al. (2023) and Muhaimin et al. (2026) similarly report praxis-dominant organizations in Kurikulum Merdeka mathematics materials, and Wiranugraha et al. (2025) find that students' thinking processes on the Pythagorean Theorem falter precisely where justification is left implicit. Taken together, this recent literature suggests that the praxis-logos imbalance diagnosed here reflects a broader, cross-context tendency rather than an isolated feature of a single textbook.

Consistent with the reframing recommended during review, the obstacles reported below are best understood as didactical hypotheses derived from the textbook's praxeological organization rather than empirically observed student difficulties. They separate obstacles attributable to textbook design choices (didactic) from those inferred from the nature of the content (epistemological) and those hypothesized from cognitive-development literature (ontogenetic), and each warrants empirical validation through clinical interviews, think-aloud protocols, classroom observation, and analysis of student work.

The integrated analysis of didactical hypotheses within the praxeological organization identifies 16 potential obstacles distributed across the three categories proposed by Brousseau (1997). Five epistemological obstacles are identified. These include the conceptual leap from geometric representation, such as square areas, to algebraic representation expressed as $c^2 = a^2 + b^2$, which according to Duval (2006) constitutes a conversion between registers that is a major source of learning difficulty. The implicit emergence of irrational numbers represents another obstacle, historically associated with foundational crises in mathematics (Kidron, 2018). Additional obstacles include the lack of elaboration on why the theorem applies only to right triangles, the unclear distinction between a theorem and its converse, and the limited variation in triangle orientation, which restricts students' ability to identify the hypotenuse.

Six didactic obstacles are directly related to the choices made in the textbook's praxeological organization. These include a suboptimal sequencing of proof and procedures and premature algebraic formalization without sufficient geometric foundation, reflecting a tension

between instructional efficiency and conceptual depth. The dominance of standard triangle orientations and the lack of counterexamples contribute to fragile concept formation. The presence of an implicit didactic contract suggests that the organization of tasks unintentionally shapes students' expectations, for example the assumption that problems in this chapter always involve hidden right triangles. The limited variety of task types further reinforces the dominance of the praxis block, with approximately 84 percent of tasks operating at the applying level in Bloom's taxonomy, while tasks requiring higher order thinking remain scarce.

Five ontogenetic obstacles are associated with cognitive demands that may exceed the developmental readiness of Grade VIII students aged 13 to 14 years. These include the requirement for formal operational thinking to understand algebraic dissection proofs, which involve multiple representations, abstract variable manipulation, and deductive reasoning. Difficulties in algebraic manipulation of square roots, such as squaring decimals and simplifying radical expressions, may also arise due to insufficient prior knowledge. Spatial reasoning challenges are evident in tasks requiring recognition of right triangles in non standard contexts and the construction of auxiliary lines. Cognitive load issues are present in multi step problems that demand the processing of several intermediate values, potentially exceeding working memory capacity. Finally, assumptions about prerequisite knowledge, including area concepts, algebraic expansion, and Cartesian coordinates with negative quadrants, may not hold for all students. Table 4 presents a summary of the identified potential learning obstacles, including their level of severity and theoretical basis.

Table 4. Summary of Hypothesized Didactical Obstacles (Didactical Hypotheses)

Code	Description	Severity Level	Theoretical Basis
E1	Transition from geometry to algebra	High	Duval (2006)
E2	Implicit irrational numbers	Medium	Fischbein (1987)
E3	Right-angle condition not elaborated	High	Tall & Vinner (1981)
D1	Less optimal sequencing of proof procedures	Medium	Harel & Sowder (1998)
D2	Dominance of standard orientation	High	Clements & Battista (1992)
D3	Implicit didactical contract	High	Brousseau (1997)
D6	Limited variation of problem types	High	Bloom et al. (1956)
O1	Demand for formal reasoning in proof	High	Jean Piaget (1970)
O4	Multi-step memory load	Medium	John Sweller (1988)
E4	Unclear distinction between the theorem and its converse	Medium	Inferred from textbook analysis
E5	Limited variation in triangle orientation	Medium	Inferred from textbook analysis
D4	Lack of counterexamples	Medium	Mason et al. (2009)
D5	Premature algebraic formalization	Medium	Van Hiele (1986)
O2	Algebraic manipulation of square roots	Medium	Inferred from textbook analysis
O3	Spatial reasoning challenges	Medium	Inferred from textbook analysis
O5	Assumptions about prerequisite knowledge	Medium	Inferred from textbook analysis

The findings of this study address the three research questions posed. First, the praxeological organization of the Pythagorean Theorem in the textbook is structured into 28 task types with an eight stage scaffolding that reflects a progression from concrete to abstract, utilizing techniques that range from algorithmic and empirical-inductive to verificatory, proportional, and multi step procedures. Second, there is a systematic imbalance between a strong praxis block and a weak logos block, where technology is largely implicit and theory is entirely absent in explicit form, causing the interconnection between techniques and their justifications to become disconnected. Third, such a praxeological organization potentially gives rise to 16 learning obstacles encompassing epistemological (5), didactic (6), and ontogenetic (5) dimensions, which cumulatively may limit the development of students' relational understanding of the Pythagorean Theorem.

CONCLUSION

This study provides a praxeological diagnosis of the Pythagorean Theorem content presented in a Grade VIII mathematics textbook developed for the Kurikulum Merdeka. The analysis identified 28 task types organized through eight scaffolding stages that progress from concrete exploration to abstract mathematical application. The textbook demonstrates several strengths in the praxis block, including varied computational and contextual tasks, culturally relevant problems, exploratory activities, and the integration of digital tools such as GeoGebra and coding. However, the central finding reveals a systematic imbalance between the praxis and logos blocks. Most learning activities emphasize procedural application and calculation, while explicit mathematical justification, theoretical explanation, and proof remain limited. Technology is predominantly presented implicitly, and the theory underlying the techniques is not clearly articulated. This imbalance may encourage students to develop instrumental understanding focused on applying formulas rather than relational understanding grounded in mathematical reasoning. The identified praxeological structure also indicates several potential epistemological, didactic, and ontogenetic obstacles, including difficulties in connecting geometric and algebraic representations, distinguishing the theorem from its converse, recognizing non-standard triangle orientations, manipulating irrational numbers, and completing multi-step problems. These findings suggest that textbook developers should strengthen the logos block by incorporating alternative proofs, explicit justification of techniques, counterexamples, varied representations, and tasks that require reasoning, argumentation, and generalization. Teachers should complement the textbook with proof discussions, conceptual reflection, multiple-solution activities, and explicit connections between procedures and underlying principles. Curriculum designers should also establish textbook quality standards that balance procedural competence with conceptual and theoretical understanding. Nevertheless, this study was limited to one chapter of a single national textbook, and the identified learning obstacles remain theoretical rather than empirically confirmed. Therefore, future research should compare multiple textbooks, extend the analysis to other mathematical topics, and investigate whether the predicted obstacles occur in actual classroom learning.

REFERENCES

- Artigue, M. (2002). Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work. *International Journal of Computers for Mathematical Learning*, 7(3), 245–274. <https://doi.org/10.1023/A:1022103903080>
- Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on the teacher's practice: The case of limits of functions in Spanish high schools. *Educational Studies in Mathematics*, 59(1), 235–268.
- Bell, A. W. (1976). A study of pupils' proof-explanations in mathematical situations. *Educational Studies in Mathematics*, 1, 23–40. <https://doi.org/https://www.jstor.org/stable/3481809>
- Bosch, M., Chevallard, Y., & Gascón, J. (2006). Science or magic? The use of models and theories in didactics of mathematics. *European Research in Mathematics Education IV*, 1254–1263.
- Bosch, M., & Gascón, J. (2006a). Twenty-five years of the didactic transposition. *ICMI Bulletin*, 58(1), 51–65. <https://shorturl.at/mtLMQ>
- Bosch, M., & Gascón, J. (2006b). *Twenty-five years of the didactic transposition*. ICMI bulletin.
- Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. Kluwer.
- Chaachoua, H., & Comiti, C. (2010). L'analyse du rôle des manuels dans l'approche anthropologique. *Apports de La Théorie Anthropologique, Diffuser Les Mathématiques (et Les Autres Savoirs) Comme Outils et Connaissances d'action*, 771–790.
- Chevallard, Y. (1989). On didactic transposition theory: some introductory notes. *International Symposium on Selected Domains of Research and Development in Mathematics Education*, 51–62. http://yves.chevallard.free.fr/spip/spip/article.php3?id_article=122

- Chevallard, Y. (1999). El análisis de las prácticas docentes en la teoría antropológica de lo didáctico. *Recherches En Didactique Des Mathématiques*, 19(2), 221–266.
- Chevallard, Y. (2019). Introducing the anthropological theory of the didactic. *Hiroshima Journal of Mathematics Education*, 12, 71–114. <https://doi.org/10.24529/hjme.1205>
- Chevallard, Y., & Bosch, M. (2020). *Didactic Transposition in Mathematics Education BT - Encyclopedia of Mathematics Education* (S. Lerman (ed.); pp. 214–218). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_48
- Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 420–464). NCTM.
- de Villiers, M. R. (2005). Three approaches as pillars for interpretive information systems research: development research, action research and grounded theory. *Proceedings of the 2005 Annual Research Conference of the South African Institute of Computer Scientists and Information Technologists on IT Research in Developing Countries*, 142–151.
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61(1–2), 103–131. <https://doi.org/10.1007/s10649-006-0400-z>
- Fischbein, E., Jehiam, R., & Cohen, D. (1995). The concept of irrational numbers in high-school students and prospective teachers. *Educational Studies in Mathematics*, 29(1), 29–44. <https://doi.org/10.1007/BF01273899>
- Florio, E. (2020). A Synergy between History of Mathematics and Mathematics Education: A Possible Path from Geometry to Symbolic Algebra. In *Education Sciences* (Vol. 10, Issue 9, p. 243). <https://doi.org/10.3390/educsci10090243>
- González-Martín, A. S., Bloch, I., Durand-Guerrier, V., & Maschietto, M. (2014). Didactic situations and didactical engineering in university mathematics: Cases from the study of Calculus and proof. *Research in Mathematics Education*, 16(2), 117–134. <https://doi.org/10.1080/14794802.2014.918347>
- Hardy, N. (2009). Students' perceptions of institutional practices: the case of limits of functions in college level Calculus courses. *Educational Studies in Mathematics*, 72(3), 341–358. <https://doi.org/10.1007/s10649-009-9199-8>
- Hendriyanto, A., Suryadi, D., Dahlan, J. A., & Juandi, D. (2023). Praxeology review: Comparing Singaporean and Indonesian textbooks in introducing the concept of sets. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(2), 1–13. <https://doi.org/10.29333/ejmste/12953>
- Huang, R., & Leung, F. K. S. (2002). How Pythagoras' theorem is taught in Czech Republic, Hong Kong and Shanghai: A case study. *Zentralblatt Für Didaktik Der Mathematik*, 34(6), 268–277. <https://doi.org/10.1007/BF02655725>
- Inhelder, B., & Piaget, J. (1958). The growth of logical thinking: From childhood to adolescence. In A. Parsons & S. Milgram (Eds.), *The growth of logical thinking: From childhood to adolescence*. (pp. xxvi, 378–xxvi, 378). Basic Books. <https://doi.org/10.1037/10034-000>
- Kemendikbudristek. (2022). *Capaian Pembelajaran Kurikulum Merdeka*.
- Kidron, I. (2018). Students' Conceptions of Irrational Numbers. *International Journal of Research in Undergraduate Mathematics Education*, 4(1), 94–118. <https://doi.org/10.1007/s40753-018-0071-z>
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics* (National r). National Academy Press.
- Krippendorff, K. (2018). *Content analysis: An introduction to its methodology*. Sage publications.
- Laborde, C. (2001). *The Use of New Technologies as a Vehicle for Restructuring Teachers' Mathematics BT - Making Sense of Mathematics Teacher Education* (F.-L. Lin & T. J. Cooney (eds.); pp. 87–109). Springer Netherlands. https://doi.org/10.1007/978-94-010-0828-0_5
- Leikin, R. (2014). *Challenging Mathematics with Multiple Solution Tasks and Mathematical Investigations in Geometry BT - Transforming Mathematics Instruction: Multiple Approaches*

- and Practices* (Y. Li, E. A. Silver, & S. Li (eds.); pp. 59–80). Springer International Publishing. https://doi.org/10.1007/978-3-319-04993-9_5
- Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, 21(2), 10–32. <https://doi.org/10.1007/BF03217543>
- Muhaimin, Lukman Hakim, Suryadi, Didi, Dasari, Dadan, Hendriyanto, Agus, & Bagus Pramatha, I Nyoman. (2026). A comparative praxeological analysis of quadrilateral and triangle concepts in Indonesian and Japanese elementary school textbooks. *Research in Comparative and International Education*, 21(1), 22–47. <https://doi.org/10.1177/17454999251411410>
- Muhaimin, L. H., Dasari, D., Hendriyanto, A., Andriatna, R., & Sahara, S. (2025). Can Augmented Reality Enhance Students' Mathematical Literacy? A Study on Technological Development for Learning Practice. *International Journal of Mathematical Education in Science and Technology*, 57(5), 1–34. <https://doi.org/10.1080/0020739X.2025.2502398>
- Muhaimin, L. H., Dasari, D., & Kusumah, Y. S. (2023). Numeracy-Ability, Characteristics of Pupils in Solving the Minimum Competency Assessment. *Jurnal Program Studi Pendidikan Matematika*, 12(1), 697–707. <https://doi.org/https://doi.org/10.24127/ajpm.v12i1.6396>
- Skemp, R. R. (2020). Relational Understanding and Instrumental Understanding. *Mathematics Teaching in the Middle School*, 12(2), 88–95. <https://doi.org/10.5951/mtms.12.2.0088>
- Sowder, L., & Harel, G. (1998). Types of students' justifications. *Mathematics Teacher*, 91(8), 670–675. <https://doi.org/https://doi.org/10.5951/MT.91.8.0670>
- Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151–169. <https://doi.org/10.1007/BF00305619>
- Treffers, A. (1993). Wiskobas and Freudenthal realistic mathematics education. *Educational Studies in Mathematics*, 25(1), 89–108. <https://doi.org/10.1007/BF01274104>
- Van Hiele, P. . (1986). *Structure and insight*. Academic Press.
- Wijayanti, D., & Winslow, C. (2017). Mathematical practice in textbooks analysis: Praxeological reference models, the case of proportion. *Journal of Research in Mathematics Education*, 6(3), 307–330. <https://doi.org/10.17583/redimat.2017.2078>
- Winsløw, C. (2011). Anthropological theory of didactic phenomena: Some examples and principles of its use in the study of mathematics education. *Accelerating the World's Research.*, 10(1), 117–138.
- Wiranugraha, M. E., Theis, R., & Iriani, D. (2025). Analysis of Students' Thinking Processes in Solving Mathematical Problems Based on Bruner's Theory in Pythagorean Theorem Tasks. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, 5(4), 1589–1609. <https://doi.org/10.51574/kognitif.v5i4.3916>
- Zazkis, R., & Leikin, R. (2008). Exemplifying definitions: a case of a square. *Educational Studies in Mathematics*, 69(2), 131–148. <https://doi.org/10.1007/s10649-008-9131-7>